Bending of Acoustic Rays in a Spherically Symmetric Field – A Comparison with Sjödin's Approach *

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The formula for the bending of acoustic rays in a fluid having a spherically symmetric inhomogeneity has been obtained without making use of Bouger's theorem. The resulting formula happens to be qualitatively similar to that obtained by Sjödin for the bending of light rays in an inhomogeneous ether. However, a particular difficulty (with a plausible way of resolution) in connection with the quantitative evaluation of the bending of acoustic rays has also been pointed out. In addition, some comments are made regarding the advantages of the adopted approach.

1. Introduction

Recently Sjödin [1] has demonstrated how the bending (and also retardation) of light rays in a central field produced by a point mass can be obtained by simply using the laws of physical optics. Such possibilities were first envisaged, as also remarked in [1], by Podlaha [2] in the context of interpreting gravitation as an effect of inhomogeneities in physical vacuum. This interpretation allows one to understand the relativistic effects as 'real effects' in line with the so-called Lorentz-Larmor ether theory. Meanwhile, we have already shown elsewhere [3] how this viewpoint, when suitably adopted in linearized aerodynamics where the role of the speed of light is known to be played by that of acoustic signals, leads consistently to certain wellknown results.

In the context of [1], therefore, one feels curious to investigate the bending of acoustic rays in a fluid with a spherically symmetric distribution of inhomogeneities. In this paper, we wish to address ourselves to this task. In spite of adopting a different approach more relevant to acoustics, we find that the desired formula is analogous to that of [1]. Finally, some comments are made regarding the advantages of the approach adopted in this paper.

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2. Bending of Acoustic Rays

Consider an acoustic beam of finite aperture traversing an inhomogeneous fluid. We should expect that the amplitude and the surfaces of constant phase of this wave are complicated functions of position. Nevertheless, as an ansatz to the wave equation

$$\nabla^2 p - \frac{1}{a^2} \frac{\partial^2 p}{\partial t^2} = 0 , \tag{1}$$

where $a\,(x, y, z)$ is the speed of sound, we may take

$$p = A\,(x, y, z) \exp i\,w \left\{ t - \frac{P\,(x, y, z)}{a_0} \right\} , \tag{2}$$

in which A has units of pressure, P has units of length and a_0 is an arbitrary reference speed. This speed can be conveniently identified with the speed of sound in the absence of inhomogeneities. Substituting (2) into (1), we get

$$\frac{\nabla^2 A}{A} - \left(\frac{w}{a_0}\right)^2 \nabla P \cdot \nabla P + \left(\frac{w}{a}\right)^2 - \frac{i\,w}{a_0}\left(\frac{2 \nabla A\, \nabla P}{A} + \nabla^2 P\right) = 0 . \tag{3}$$

If A and ∇P vary slowly enough such that

$$|A^{-1} \nabla^2 A| \ll (w/a)^2 , \quad |\nabla^2 P| \ll w/a$$

and

$$|A^{-1} \nabla A \cdot \nabla P| \ll w/a ,$$

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then (3) simplifies to the wellknown eikonal equation

$$\nabla P \cdot \nabla P = \varrho^2 , \tag{4}$$

where

$$\varrho (x, y, z) = a_0/a \tag{5}$$

is the refractive index. The above conditions physically imply that the waves described by (2) are plane waves over small regions of space. Furthermore, ∇P defines, point by point, the direction of travel of each acoustic ray so that the solution of (4) provides the trajectories of the ray paths traversed by the acoustic energy.

Without further loss of generality, let us take

$$\nabla P = \varrho (x, y, z)\, \hat{e} (x, y, z) , \tag{6}$$

where $\hat{e}$ is the unit vector along the ray. Introducing the distance parameter s along the ray path, the changes in ∇P can be determined from the vector differential $\mathrm{d}/\mathrm{d}s\,(\nabla P)$. Computing, component by component, one finds for the choice of (6) that

$$\frac{\mathrm{d}}{\mathrm{d}s} (\nabla P) = \nabla \varrho . \tag{7}$$

Now assume, in accordance with [1],

$$\varrho (x, y, z) = h + k\, r^{-1} = h + k\, (x^2 + y^2 + z^2)^{-1/2} , \tag{8}$$

but leave the constants h and k unspecified for the moment for reasons indicated later. Thus

$$|\nabla \varrho| = |-k\, r^{-2}| . \tag{9}$$

Hence, the total angular deviation $\delta\varphi$ which the vector ∇P suffers for a ray coming from minus infinity and going to plus infinity is given by [1]

$$\delta\varphi = -2 \int_{r_0}^{\infty} k\, r^{-2}\, \mathrm{d}r = -2k\, r_0^{-1} , \tag{10}$$

where, as usual, $r_0\ (\neq 0)$ is the minimum distance between the ray and the center of the spherical field. Notice that the result (10) is *qualitatively* the same as that (to the first order) obtained in [1], viz. that the total deviation $\delta\varphi$ is inversely proportional to the distance r_0. Curiously, it appears that the form of $\delta\varphi$ is dictated by the choice of $\varrho(r)$. For example, one may start with the expression

$$\varrho(r) = h + k\, r^{-1} + l\, r^{-2} + m\, r^{-3} + \ldots \tag{11}$$

to obtain

$$\delta\varphi = -2k\, r_0^{-1} + l'\, r_0^{-2} + m'\, r_0^{-3} + \ldots \tag{12}$$

in higher order approximations.

3. Comments

Although the foregoing treatment verbally refers only to acoustic rays, it is clear that the same procedure can, in principle, be adopted also for light rays in inhomogeneous ether. In addition, this alternative derivation, based on wave approach, elucidates the underlying physical process by which the bending (of rays) takes place. More importantly, however, notice that no use is made of Bouger's theorem [4] so that one remains *free to choose* the form of $\varrho(r)$ till the end. In the same token, we bypassed another complexity, namely, that of evaluating the integral in (3) of [1]. Note that the choice of (11), even with finite number of higher order (>1) terms, renders that integral so complicated that a solution in a closed form may not be obtainable.

The freedom of choice referred to above is consistent with the fact that in a acoustic (e.g., fluid) medium, one encounters more commonly non-spherical complicated distributions (at least locally). Indeed, as correctly remarked in [1], a more appropriate choice would be a time-dependent ϱ, viz. $\varrho(r, t)$ instead of mere $\varrho(r)$. For heuristic motives, however, the present choice (11) serves allright.

The constants $h, k, l, m, \ldots$ were left unspecified (in contrast to the specification in [1], viz. of $k = 2MG/c_0^2$) for the reason that the presence of a point mass M can not be held responsible (at least in a manner prescribed in [1]) for *causing* permanent inhomogeneities out in the fluid. In the present context, therefore, it seems reasonable that these constants are determined by state variables in some way; although it is a matter of further research to demonstrate exactly how. However, as a plausible resolution to this difficulty (viz. in the quantitative estimation of $\delta\varphi$), the author tends to accept those constants as kinematical parameters *uniquely* char-

[1] It is instructive to obtain the result (10) directly from (7) by writing it out in polar coordinates. Bouger's theorem then *follows* as a by-product.

acterizing each particular state of the medium in the following sense.

If one is interested, for example, to retain in (11) terms up to second order, one has to perform two initial trial measurements estimating two sets of values $(r_0', \delta\varphi')$ and $(r_0'', \delta\varphi'')$ so that (12) yields

$$-2k = (r_0''^2\, \delta\varphi'' - r_0'^2\, \delta\varphi')\,(r_0'' - r_0')^{-1}\,, \tag{13}$$

$$l' = r_0' r_0''\,(r_0'\,\delta\varphi' - r_0''\,\delta\varphi'')\,(r_0'' - r_0')^{-1}, \quad r_0'' \neq r_0'.$$

These numerical values, once fixed, can be reasonably used for measuring $\delta\varphi$ in general so long as the state of the medium does not change. Note also that $\delta\varphi \to 0$ only if $r_0 \to \infty$.

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